Problem 34

In each of Problems 34 through 37, construct a first order linear differential equation whose solutions have the required behavior as $t \to \infty$. Then solve your equation and confirm that the solutions do indeed have the specified property.

All solutions have the limit 3 as $t \to \infty$.

Solution

The rate of change of y will become negligible as t gets big enough, so we choose

$$y' + y = 3.$$

This is a first-order linear inhomogeneous ODE, so it can be solved by multiplying both sides by an integrating factor I.

$$I = \exp\left(\int^t 1\,ds\right) = e^t$$

Proceed with the multiplication.

$$e^t y' + e^t y = 3e^t$$

The left side can be written as $d/dt(Iy)$ using the product rule.

$$\frac{d}{dt}(e^t y) = 3e^t$$

Integrate both sides with respect to t.

$$e^t y = \int^t 3e^s \, ds + C$$
$$= 3e^t + C$$

Divide both sides by e^t to obtain the general solution for y.

$$y(t) = 3 + \frac{C}{e^t}$$

Take the limit of both sides as $t \to \infty$.

$$\lim_{t\to\infty} y(t) = \lim_{t\to\infty} 3 + \underbrace{\lim_{t\to\infty} \frac{C}{e^t}}_{=\,0} = 3$$

Therefore, all solutions of $y' + y = 3$ have the limit 3 as $t \to \infty$.